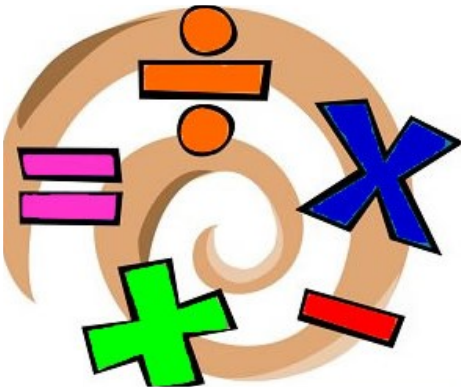
Four Rules Guide for Parents



A guide for parents on how we teach formal written methods of calculation in mathematics for the four rules of number - addition, subtraction, multiplication and division

Introduction

This guide is intended to help parents in understanding the methods, sequencing of teaching and progression of written methods relating to the four rules of calculation – addition, subtraction, multiplication and division that we teach at Eversley Primary School.

We recognise that children progress at different rates. Some children will move quickly through these stages, becoming confident in calculating numbers quickly. Others will need more time to consolidate each step before moving on. Our aim is that all pupils by the time they leave us here at Eversley can confidently and competently use a method to add, subtract, multiply and divide numbers so they are maths-ready for secondary school and the world of work.

How we teach calculation at Eversley Primary School

We begin building the foundations in the understanding of and skills necessary for calculation in EYFS and Key Stage 1 by helping children understand the abstract nature of number through concrete, hands-on experience of number, using a range of practical equipment, such as:

- Numicon
- objects to count (counters, toys, multilink cubes etc)
- bead strings
- marked and unmarked numberlines
- Dienes materials
- arrow cards
- hundred squares
- multiplication squares
- counting sticks
- arrays
- exchange boards
- maths ITPs (interactive teaching programmes) on interactive whiteboards

Before moving children on to informal and formal written methods of calculation we ensure that children have a solid understanding of the relationship between operations: that addition/ subtraction and multiplication/division are inverse operations to one another and that multiplication can be seen as repeated addition and division as repeated subtraction.

We ensure that children have practised and developed a range of mental calculation strategies and are secure in their knowledge of number bonds up to 10, then 20 and with multiples of 10 up to 100 and can recall quickly and out of sequence multiplication facts up to 12x12 and the corresponding division facts.

Addition

Vocabulary to be used when teaching addition:

add plus addition

and + sum

total increase

altogether greater than more than

Pre-requisite skills and knowledge needed before formal written methods for addition are taught

By the end of Key Stage 1:

- recall key number facts instantly – for example, all addition facts for each number to 10 then to 20, and sums of multiples of 10;
- appreciate that addition and subtraction are the inverse of each other;
- recognise that addition can be done in any order and use this to add mentally a one-digit number or a multiple of 10 to any one-digit or two-digit number;
- partition two-digit numbers in different ways (for example, 37 = 30 + 7 and 20 + 17) and add the tens and ones separately, then recombine.

When these abilities are secure, children can be taught the efficient columnar addition method and will be able to carry them out efficiently and accurately.

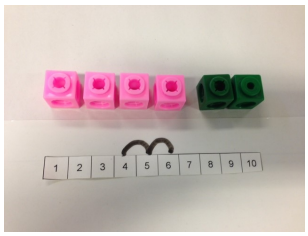
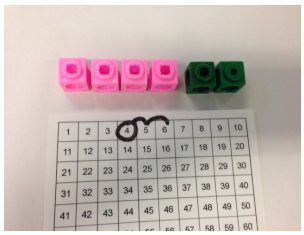
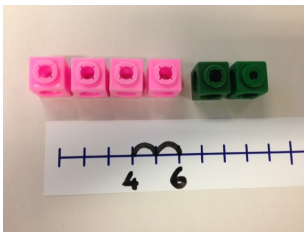
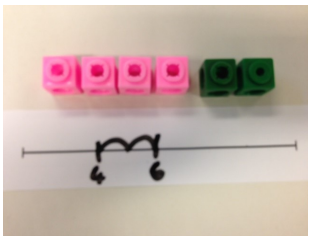
Progression in teaching columnar addition:

- adding a pair (or more) of two-digit numbers, without carrying
- adding a pair (or more) of two-digit numbers, carrying ones
- adding a pair (or more) of two-digit numbers, carrying tens
- adding a pair (or more) of two-digit numbers, carrying ones and tens
- adding a pair (or more) of three-digit numbers or a three-digit number to a two-digit number, carrying into hundreds
- adding a pair (or more) of four-digit or greater numbers together, using carrying across some or all place values
- adding a pair (or more) of decimals together, ensuring digits lined up in place values, with carrying between decimal values and across the decimal point to ones.

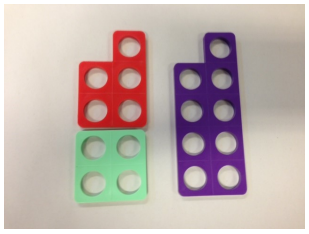
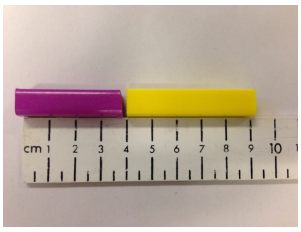
Addition

STAGE 1	***Year 1***

Count on using a variety of resources, such as numberlines and number squares:

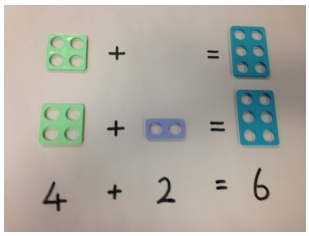





Develop concept of number bonds, initially to 10, then 20.
Record related number facts e.g. 4 + 5 = 9, 5 + 4 = 9, 9 = 4 + 5, 9 = 5 + 4

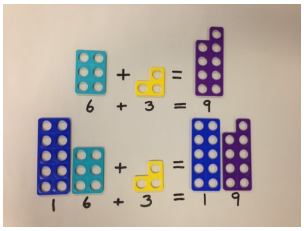
STAGE 2	***Year 1***

Develop understanding of the + and = symbols and the concept of 'empty box' calculations
e.g. 4 + ☐ = 6



Use understanding of patterning, place value and partitioning to derive number facts
e.g. 6 + 3 = 9 (known fact)
16 + 3 = 19
26 + 3 = 29

Begin to use this understanding of place value and partitioning to carry out addition of one-digit and two-digit numbers, recording workings as horizontal calculations:



STAGE 3	***Year 2***

Continue to develop understanding of partitioning and place value and use this to support adding with TU and O:

41 + 8 =
40 + 1 + 8 =
40 + 9 = 49

Practical apparatus can continue to support this, such as numberlines and hundred squares, recording the calculations as a horizontal number sentence.

41 + 8 = 49

When confident with the concept of partitioning, place value and horizontal recording children can move on to columnar recording of workings, using calculations where no exchange is necessary:

41 + 8 =

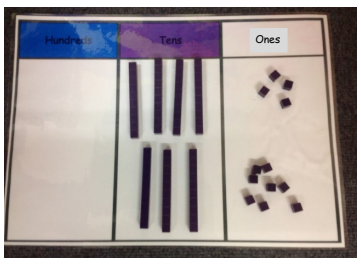
$$\begin{array}{rr} & 4\ \ 1 \\ + & \quad 8 \\ \hline & 4\ \ 9 \\ \hline \end{array}$$

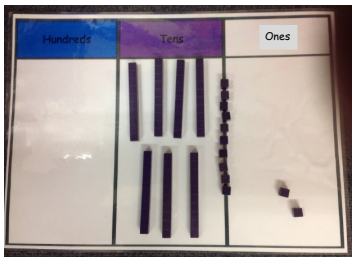
STAGE 4	***Year 3***

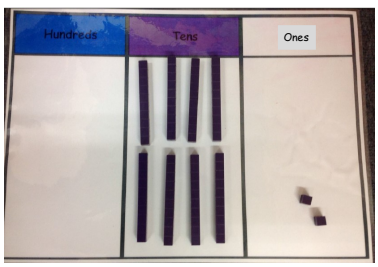
Once children are confident with setting their work out in columns, children can be introduced to the concept of exchange, supporting their working with exchange boards:

44 + 38 =

$$\begin{array}{rr} & 4\ \ 4 \\ + & 3\ \ 8 \\ \hline & 8\ \ 2 \\ \hline & {\scriptstyle 1} \end{array}$$







Children can then move onto adding increasingly bigger numbers and use numbers where exchange occurs more than once:

257 + 129 =

	2	5	7
+	1	2	9
	3	8	6
		1	

375 + 248 =

	3	7	5
+	2	4	8
	6	2	3
	1	1	

Once secure children can move onto larger numbers, adding more than two numbers together and adding decimals, beginning with decimal numbers in the context of money and measures:

2643 + 3987 =

	2	6	4	3
+	3	9	8	7
	6	6	3	0
	1	1	1	

2.654 + 3.88 =

	2	·	6	5	4
+	3	·	8	8	0
	6	·	5	3	4
	1		1		

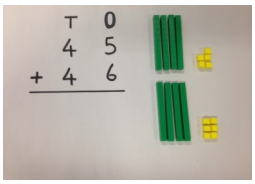
183 + 79 + 82 =

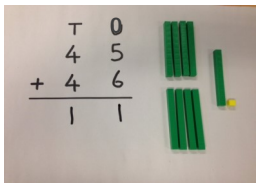
	1	8	3
		7	9
+		8	2
	3	4	4
	2	1	

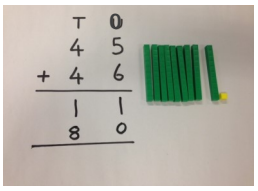
SHOULD CHILDREN STRUGGLE WITH STAGE 5, ALLOW CHILDREN TO USE MORE EXPANDED METHODS BEFORE RETURNING TO COMPACT METHODS:

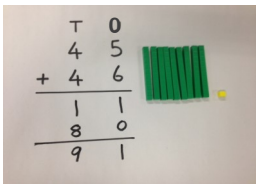
45 + 46 =

	4	5
+	4	6
	1	1
	8	0
	9	1









Subtraction

Vocabulary to be used when teaching subtraction:

subtract	take away	subtraction
minus	–	difference between
decrease	fewer than	less than

Pre-requisite skills and knowledge needed before formal written methods for subtraction are taught

By the end of Key Stage 1:

- recall key number facts instantly – for example, all addition and subtraction facts for each number to 10 then to 20, differences of multiples of 10;
- appreciate that addition and subtraction are the inverse of each other;
- partition two-digit numbers in different ways (for example, 37 = 30 + 7 and 20 + 17);
- understand that when subtracting, the biggest number is written first in the written horizontal layout (for example: 37 - 19 = 18).

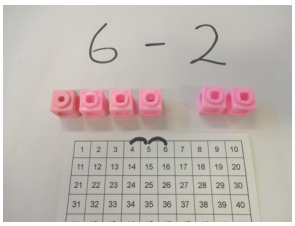
Progression in teaching columnar subtraction:

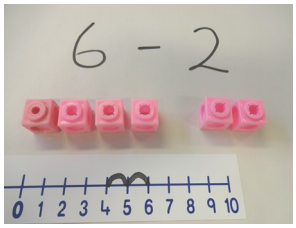
- subtracting a single-digit or two-digit number from a two-digit number, without borrowing
- subtracting a single-digit or two-digit number from a two-digit number, borrowing from tens
- subtracting a two-digit or three-digit number from a three-digit number, borrowing from hundreds
- subtracting a two-digit or three-digit number from a three-digit number, borrowing from tens and hundreds
- subtraction which will require borrowing through a zero e.g. 502 - 78
- subtracting four-digit or greater numbers from a four digit number or greater, using borrowing across some or all place values, including through a zero
- subtracting a number that will require borrowing across a series of zeroes e.g. 2003 - 467
- subtracting decimals, ensuring digits lined up in place values, with borrowing between decimal values and across the decimal point from ones, including through a zero

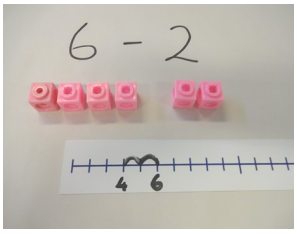
Subtraction

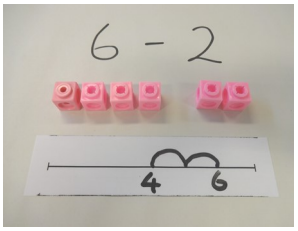
STAGE 1	***Year 1***

Count back using a variety of resources, such as numberlines and number squares:

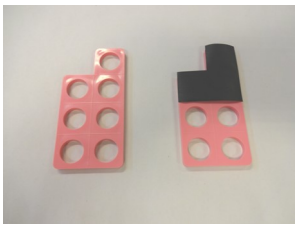








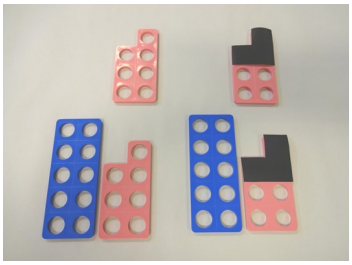
Develop subtraction facts, initially to 10, then 20.
Record related number facts e.g. 7 - 3 = 4 and 7 - 4 = 3 and make links to related addition facts e.g. 7 – 3 = 4 so 4 + 3 = 7 and 3 + 4 = 7.



STAGE 2	***Year 1***

Develop understanding of the - and = symbols and the concept of 'empty box' calculations
e.g. 9 - ☐ = 5

Use understanding of patterning, place value and partitioning to derive number facts
e.g. 7 - 3 = 4 (known fact)
17 - 3 = 14
27 - 3 = 24



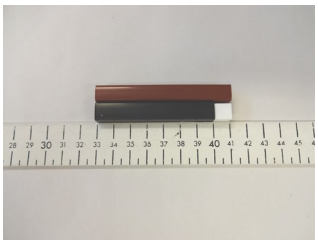
Begin to use this understanding of place value and partitioning to carry out subtraction of a one-digit number from a two-digit number, recording workings as horizontal calculations.

STAGE 3	***Year 2***

Continue to develop understanding of partitioning and place value and use this to support subtraction with TU and O:

41 - 8 =
41 - 1 - 7 =
40 - 7 = 33

Practical apparatus can continue to support this, such as numberlines and hundred squares, recording the calculations as a horizontal number sentence.



41 - 8 = 33

STAGE 4	***Year 2***

Horizontal recording can begin to be replaced with recording in columns without the need for exchange:

48 - 7 =

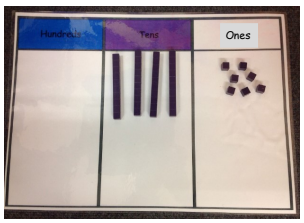
$$\begin{array}{r} 4\ 8 \\ -\ \ 7 \\ \hline 4\ 1 \\ \hline \end{array}$$

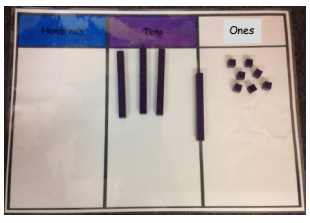
STAGE 5	***Year 3***

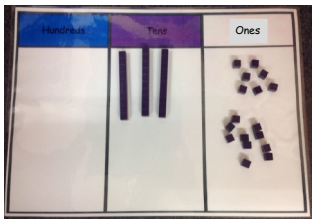
Using exchange boards for support, children can then begin to record in columns subtraction calculations that require exchange:

47 - 8 =

$$\begin{array}{r} {\scriptstyle 3} \\ \not{4}\ {}^{1}7 \\ -\ \ 8 \\ \hline 3\ 9 \\ \hline \end{array}$$







STAGE 6 **Year 3 onwards**

Extend the use of the column method for subtraction requiring more than one exchange and where 0 is a place holder, continuing to support with exchange boards if necessary:

435 - 268 =

	3	12	
	~~4~~	~~3~~	15
-	2	6	8
	1	6	7

502 - 278 =

	4	9	
	~~5~~	10	12
-	2	7	8
	2	2	4

2001 - 546 =

	1	9	9	
	~~2~~	10	10	11
-		5	4	6
	1	4	5	5

SHOULD CHILDREN STRUGGLE WITH STAGE 5 or 6, ALLOW CHILDREN TO USE MORE EXPANDED METHODS BEFORE RETURNING TO COMPACT METHODS:

Without exchange:

48 - 7 =

	4	8
-		7

	4	0	+	8		
-				7		
	4	0	+	1	=	41

With exchange:

47 - 8 =

	4	7
-		8

	3	0	+	1	7	
-					8	
	3	0	+		9	= 39

45 - 26 =

	4	5
-	2	6

	4	0	+	5		
-			+	6		

	3	0	+	1	5	
-	2	0	+		6	
	1	0	+		9	= 19

Multiplication

Vocabulary to be used when teaching multiplication:

multiply | times | multiplication

product | × | groups of

lots of | double

Pre-requisite skills and knowledge needed before formal written methods for multiplication are taught

By the end of lower Key Stage 2:

- recall all multiplication facts to 12 × 12;
- partition numbers into hundreds, tens and ones;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 from 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method.

When these abilities are secure, children can be taught informal and formal written methods for multiplication and will be able to carry them out efficiently and accurately.

Progression in teaching informal and formal written methods for multiplication:

- multiply a two-digit number by a one-digit number (including zero as a place value)
- multiply a three-digit number or greater by a one-digit number (including zero as a place value)
- multiply a two-digit number by a two-digit number (including zero as a place value)
- multiply a three-digit number or greater by a two-digit number (including zero as a place value)
- multiply a decimal number by a one-digit or two-digit number

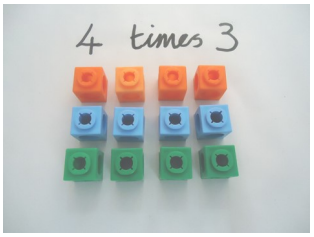
Multiplication

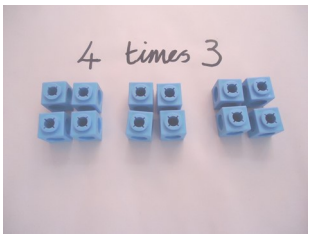
Most formal written methods for multiplication are introduced in Key Stage 2. In order to help pupils use these methods effectively, by the end of Key Stage 1 pupils need to be confident in seeing multiplication as repeated addition, using arrays to depict multiplication and are becoming more confident in knowing their multiplication facts. The expectation is that children are secure in their knowledge of times table facts, up to 12 x 12, by the end of Year 4.

Children that are less confident with their times table facts should use numbers they are confident with so they can still practise and understand the written methods.

STAGE 1	***Year 1***

Use practical apparatus and diagrams to understand multiplication as repeated addition:

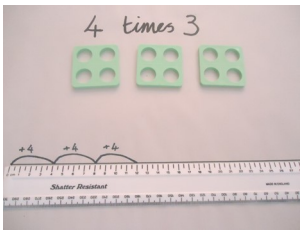




4 x 3 = 12

STAGE 2	***Year 1/2***

Use numberlines to show multiplication as repeated addition:



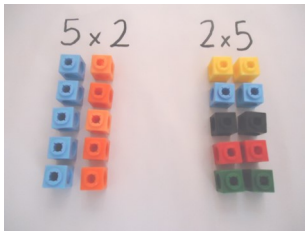
4 x 3 = 12

STAGE 3	***Year 2***

Develop use of x and = symbols to record multiplication calculations horizontally.

Use arrays and other practical apparatus to illustrate commutativity (that multiplication calculations can be carried out in any order) e.g. 2 x 5 arrives at the same product as 5 x 2.

Begin to derive new facts from known facts
e.g. 3 x 2 = 6 (known fact)
30 x 2 = 60
300 x 2 = 600



STAGE 4 **Year 3**

Using times table facts that children are confident with, children should begin using the compact method for multiplying:

27 x 4 =

```
  2 7
x   4
-------
  1 0 8
-------
  2
```

329 x 5 =

```
    3 2 9
x       5
---------
  1 6 4 5
---------
  1 1
```

STAGE 5 **Year 4**

Children should move onto the compact method for multiplying by a two digit number. Both the expanded or contracted form will be acceptable formal methods:

124 x 15 =

```
    1 2 4
x     1 5
---------
  1 8 6 0
---------
  1 3
```

or

124 x 15 =

```
    1 2 4
x     1 5
---------
    6 2 0
  1 1
1 2 4 0
---------
1 8 6 0
---------
```

Add 0 before multiplying by the number in the tens value.

124 x 35 =

```
    1 2 4
x     3 5
---------
  4 3 4 0
---------
  1
```

or

124 x 35 =

```
    1 2 4
x     3 5
---------
    6 2 0
  1 2
3 7 2 0
  1
---------
4 3 4 0
---------
1
```

When multiplying by a 'small' two-digit number, e.g. 12 or 13 or tens numbers, it is expected that children can develop strategies to calculate these mentally.

SHOULD CHILDREN STRUGGLE WITH STAGES 4 and 5, ALLOW CHILDREN TO USE MORE EXPANDED METHODS BEFORE RETURNING TO COMPACT METHODS:

27 x 8 =

x	20	7
8	160	56

```
  1 6 0
+   5 6
-------
  2 1 6
  1
```

346 x 6 =

x	300	40	6
6	1800	240	36

```
  1 8 0 0
    2 4 0
+     3 6
---------
  2 0 7 6
  1 1
```

34 x 46 =

x	40	6
30	1200	180
4	160	24

```
  1 2 0 0
    1 8 0
    1 6 0
+     2 4
-----------
  1 5 6 4
    1
```

Division

Vocabulary to be used when teaching division:

divide

division

shared between

÷

equal groups of

divided by

halve

Pre-requisite skills and knowledge needed before formal written methods for division are taught

By the end of lower Key Stage 2:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into hundreds, tens and ones into multiples of divisor (for example, partition 387 into multiples of 3: 300 + 60 + 27);
- recall multiplication and division facts to 12 × 12;
- understand concept of remainders, and know how to find a remainder when working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

When these abilities are secure, children can be taught informal and formal written methods for division and will be able to carry them out efficiently and accurately.

Progression in teaching informal and formal written methods for division:

- divide a two-digit number by a one-digit number, including with a remainder
- divide a three-digit number or greater by a one-digit number, including with a remainder
- divide a three-digit number or greater by a two-digit number, including with a remainder
- divide a decimal number by a one-digit or two-digit number, including with a remainder
- express remainders as fractions and decimals of divisor

Division

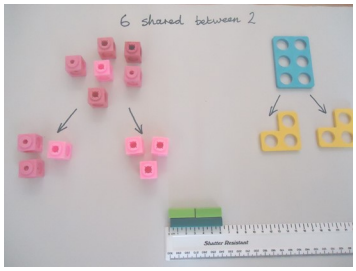
Division can be taught in the context of sharing (e.g. 12 ÷ 3 = "If there are 12 sweets how many sweets will 3 children get each?") and grouping (e.g. 12 ÷ 3 = "How many groups of 3 sweets can I make if I have 12 sweets?"). In Key Stage 1 it is important that children are familiar and have experienced in lessons both interpretations of division.

Throughout both Key Stages children should be encouraged to learn division facts alongside multiplication facts (e.g. 5 x 4 = 20, 20 ÷ 5 = 4, 20 ÷ 4 = 5). As with times table facts, children should be secure in their division facts by the end of Year 4.

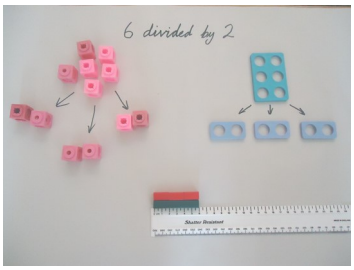
As with other methods, children should be taught in Key Stage 2 the compact methods from the beginning. Should children struggle with the compact method they should use more expanded methods.

STAGE 1	***Year 1***

Using apparatus, children to explore division as sharing:

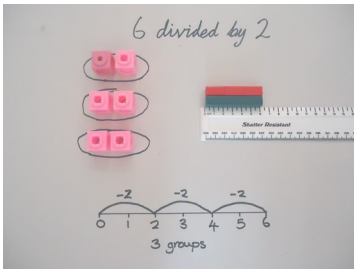


Children to develop their concept of division as also being grouping, using repeated subtraction (removing groups of the same size) using practical apparatus and diagrams:



STAGE 2	***Year 2/3***

Children to develop understanding of division using arrays and numberlines showing repeated groups:



Use numberlines to show repeated grouping (repeated subtraction of sets the same size).

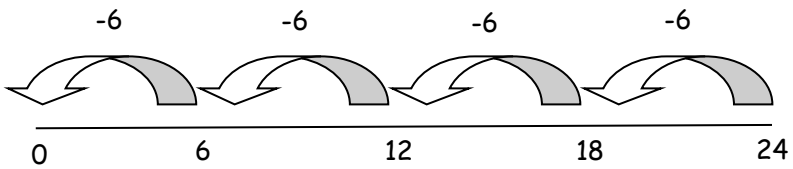
Develop use of ÷ and = symbols to record division calculations horizontally.

Use arrays and other practical apparatus to illustrate making of repeated groups.

Begin to derive new facts from known facts
e.g. 6 ÷ 3 = 2 (known fact)
60 ÷ 3 = 20
600 ÷ 3 = 200

As children begin to experience division through sharing and grouping, division as grouping can be recorded on the numberline and children can understand division as repeated subtraction:

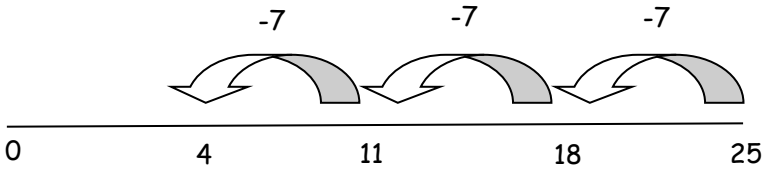
24 ÷ 6 =



Number of groups taken away is 4, therefore 24 ÷ 6 = 4

Include calculations that contain remainders:

25 ÷ 7 =



Number of complete groups taken away is 3, with 4 left over, therefore 25 ÷ 7 = 3 r 4.

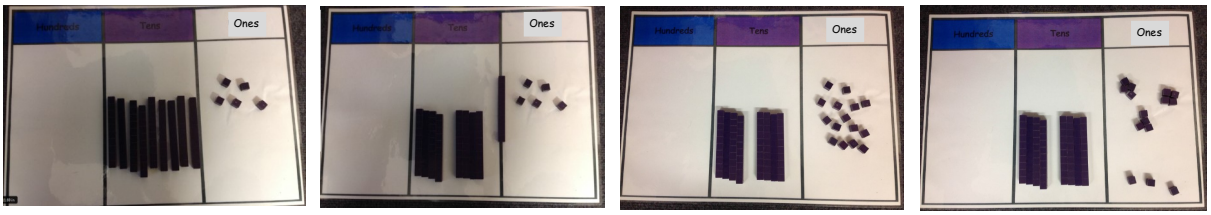
These steps can still be supported with apparatus if necessary.

STAGE 3 — ***Year 3/4***

Children should be immediately taught the short division method, but in order for children to understand the place values of number in division, exchange boards should be used:

95 ÷ 4 =

```
    2  3   r  3
  ┌──────
4 │ 9  15
```



STAGE 4 — ***Year 4 & 5***

Once children are confident in dividing using a one-digit number without support from the exchange boards, then children can move onto dividing by two-digit numbers. Before dividing, children should list multiples of the divisor up to 10x the amount. This will aid the short division process:

275 ÷ 17 =

```
      0  1   6   r  3
  ┌──────────
1 7 │ 2  27  105
```

17
34
51
68
85
102
119
136
153
170

This skill can be practised as an OMS or during Early Morning Work.

Types of short division calculations:

- no exchange, no remainder — e.g. 4) 848
- no exchange, with remainder — e.g. 3) 635
- with exchange, no remainder — e.g. 7) 994
- with exchange, with remainder — e.g. 3) 470
- empty place at start of quotient — e.g. 7) 287
- noughts in the quotient — e.g. 4) 840 or 8) 5608
- decimal dividend — e.g. 5) 61.5 or 3) 4.26

In Year 5 and 6, children can begin to express remainders as fractions and decimals:

166 ÷ 6 =

290 ÷ 8 =

$$\begin{array}{r} 0\ 3\ 6\cdot 2\ 5 \\ 8\overline{)\ 2\ 9\ {}^{5}0\cdot {}^{2}0\ {}^{4}0} \end{array}$$

290 ÷ 8 = 36.25 or $36\frac{1}{4}$

SHOULD CHILDREN STRUGGLE WITH STAGES 3 and 4, ALLOW CHILDREN TO USE MORE EXPANDED METHODS BEFORE RETURNING TO COMPACT METHODS:

Without chunking:

25 ÷ 7 =

$$\begin{array}{r} 7\overline{)\ 2\ 5} \\ -\ \ 7 \\ \hline 1\ 8 \\ -\ \ 7 \\ \hline 1\ 1 \\ -\ \ 7 \\ \hline 4 \end{array}$$

25 ÷ 7 = 3 r 4

With chunking:

166 ÷ 6 =

$$\begin{array}{r} 6\overline{)\ 1\ 6\ 6} \\ -\ \ 6\ 0 \quad (10\times 6) \\ \hline 1\ 0\ 6 \\ -\ \ 6\ 0 \quad (10\times 6) \\ \hline 4\ 6 \\ -\ \ 3\ 0 \quad (5\times 6) \\ \hline 1\ 6 \\ -\ \ 1\ 2 \quad (2\times 6) \\ \hline 4 \end{array}$$

Chunking wallet:

1 x 6 = 6
2 x 6 = 12
5 x 6 = 30
10 x 6 = 60
20 x 6 = 120

166 ÷ 6 = 27 r 4